1. Gauss - Jordan

2. complex system

$$[C]\{Z\} = \{W\}$$

$$C = A + iB$$
$$Z = X + iY$$
$$W = U + iV$$

$$[A+iB]\{X+iY\} = \{U+iV\}$$

$$[A]X + iBX + iAY - BY = U + iV$$

$$AX - BY = U$$
$$BX + AY = V$$

LU Decomposition

$[A]\{X\} - \{B\} = 0$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

$$[U]\{X\} - \{D\} = 0$$

Assume there is a lower triangular matrix with 1 on the diagonal.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

$$[L]\{[U]\{X\} - \{D\}\} = [A]\{X\} - \{B\}$$

$$\Rightarrow [L][U] = [A]$$

and

$$[L]\{D\} = \{B\}$$

Strategy.

1. LU decomposition step: A is decomposed into L & U
2. Substitution: L & U are used to determine $\{X\}$

Use forward substitution to get $\{D\}$

use backward substitution for $\{X\}$

$$A\,X = B$$

Decomposition

$U \quad L$

$$L\,D = B$$ Forward substitution

$$U\,X = D$$ — Backward substitution

Gauss Elimination

$\rightarrow$ Forward Elimination $\Rightarrow U$

$$
\begin{array}{cc}
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} &
\begin{array}{ll} \times \dfrac{a_{21}}{a_{11}} & f_{21} \\ & \\ \dfrac{a_{31}}{a_{11}} & f_{31} \end{array}
\end{array}
$$

$$
\begin{array}{cc}
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & a'_{32} & a'_{33} \end{bmatrix} &
\begin{array}{ll} \dfrac{a'_{32}}{a'_{22}} & f_{32} \end{array}
\end{array}
$$

$$
U = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix}
$$

$\cancel{U = a_{11}}$

$$
L = \begin{bmatrix} 1 & 0 & 0 \\ f_{21} & 1 & 0 \\ f_{31} & f_{32} & 1 \end{bmatrix}
$$

Algorithm

Decomposition (a, n)

```
DO  K = 1, n-1
    DO  i = K+1, n
        fac = a(i,K) / a(K,K)
        a(i,K) = fac
        DO  j = K+1, n
            a(i,j) = a(i,j) - fac * a(K,j)
        End do
    End do
End do
```

$$fac = \frac{a_{i,K}}{a_{K,K}}$$

$$a_{i,K} = fac$$

$$a_{i,j} = a_{i,j} - fac * a_{K,j}$$

Forward Substitution

$$d_i = b_i - \sum_{j=1}^{i-1} a_{i,j}\, d_j \qquad \text{for } i = 2, 3, \ldots n$$

Backward substitution

$$x_n = \frac{d_n}{a_{nn}}$$

$$x_i = \frac{d_i - \sum_{j=i+1}^{n} a_{i,j}\, x_j}{a_{ii}} \qquad \text{for } i = n-1, n-2, \ldots, 1$$

$$\frac{n^3}{3}$$