

1. Gauss - Jordan

①

2. Complex system

$$[C] \{z\} = \{w\}$$

$$C = A + iB$$

$$z = x + iy$$

$$w = u + iv$$

$$[A + iB] \{x + iy\} = \{u + iv\}$$

$$[A]x + iBx + iAy - By = u + iv$$

$$Ax - By = u$$

$$Bx + Ay = v$$

LU decomposition

②

$$[A]\{x\} = \{B\}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

$$[U]\{x\} = \{D\}$$

Assume there is a lower triangular matrix with 1 on the diagonal.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

$$[L] \{ [U]\{x\} - \{D\} \} = [A]\{x\} - \{B\}$$

$$\Rightarrow [L][U] = [A]$$

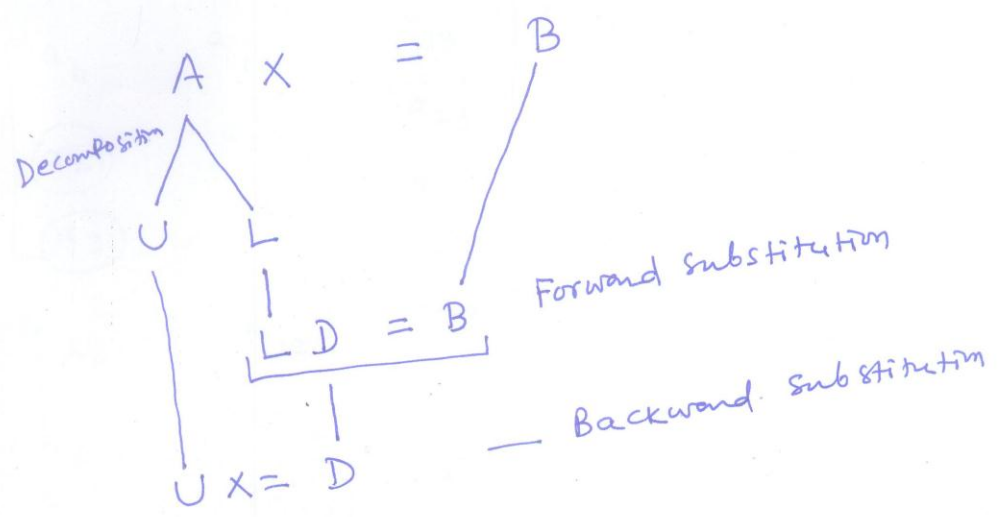
and

$$[L]\{D\} = \{B\}$$

Strategy.

1. LU decomposition step:
A is decomposed into L & U
2. Substitution: L & U are used to determine $\{x\}$

Use forward substitution to get $\{D\}$
Use backward substitution for $\{X\}$



Gauss Elimination

→ Forward Elimination ⇒ U

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \frac{a_{21}}{a_{11}} \quad f_{21}$$

$$\frac{a_{31}}{a_{11}} \quad f_{31}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & a'_{32} & a'_{33} \end{bmatrix} \frac{a'_{32}}{a'_{22}} \quad f_{32}$$

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ f_{21} & 1 & 0 \\ f_{31} & f_{32} & 1 \end{bmatrix}$$

Algorithm

Decomposition (a, n)

```

DO k = 1, n-1
  DO i = k+1, n
    fac = ai,k / ak,k
    ai,k = fac
    DO j = k+1, n
      aij = aij - fac * ak,j
    Enddo
  Enddo
Enddo

```

Forward Substitution

$$d_i = b_i - \sum_{j=1}^{i-1} a_{ij} d_j \quad \text{for } i = 2, 3, \dots, n$$

Backward Substitution

$$x_n = \frac{d_n}{a_{nn}}$$

$$x_i = \frac{d_i - \sum_{j=i+1}^n a_{ij} x_j}{a_{ii}} \quad \text{for } i = n-1, n-2, \dots, 1$$

$$\left(\frac{n^3}{3} \right)$$